



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Hence

$$\binom{n}{a} - \binom{i}{1} \binom{n-1}{a} + \binom{i}{2} \binom{n-2}{a} - \cdots + (-1)^i \binom{n-i}{a-i} = \binom{n-i}{a-i}. \quad (1)$$

In the case when $i = a$, we have $a - i = 0$, and $\binom{n-i}{a-i} = 1$; therefore, equation (1) becomes

$$\binom{n}{a} - \binom{a}{1} \binom{n-1}{a} + \binom{a}{2} \binom{n-2}{a} - \cdots + (-1)^a \binom{n-a}{a} = 1. \quad (2)$$

When $i > a$, then $a - i$ is negative, so that $\binom{n-i}{a-i} = 0$. Hence for $i > a$,

$$\binom{n}{a} - \binom{i}{1} \binom{n-1}{a} + \binom{i}{2} \binom{n-2}{a} - \cdots + (-1)^i \binom{n-i}{a} = 0. \quad (3)$$

An excellent solution by a somewhat different method from the ones exhibited in these two solutions was received from A. M. HARDING.

425. Proposed by CLIFFORD N. MILLS, Brookings, S. D.

Solve for x and y the equations $2^{x+y} = 6$, $2^{x+1} = 3^y$.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We shall agree to mean by a^x , where a is real, $e^{x \log a}$, in which the real logarithm of a is to be taken, so that a^x is single valued. We have, then, from the given equations, $(x + y) \log 2 = \log 6$, $(x + 1) \log 2 = y \log 3$, where all the logarithms are real. Solving these linear equations for x and y , we obtain the required solutions

$$x = \frac{(\log 6)^2 - \log 2 \cdot \log 12}{\log 2 \log 6} = 1.198 \text{ and } y = \frac{\log 12}{\log 2} = 1.387.$$

On account of the homogeneity of these fractions in the logarithmic function, we may substitute logarithms to the base 10 in our computation of the numerical values of x and y .

Also solved by EMMA GIBSON, NATHAN ALTSHILLER, A. L. McCARTY, A. M. HARDING, HORACE OLSON, RICHARD MORRIS, G. W. HARTWELL, C. E. GITHENS, FRANK IRVIN, ELIZABETH B. DAVIS, F. L. CARMICHAEL, EDWARD S. INGHAM, S. A. JOFFE, W. C. EELLS, ELMER SCHUYLER, V. M. SPUNAR, R. M. MATHEWS, and CYRIL A. NELSON.

GEOMETRY.

454. Proposed by LOUIS ROUILLION, Mechanics Institute, New York City.

Show how to construct an equilateral triangle with its vertices lying on three [parallel] lines not equally spaced.

I. SOLUTION BY C. N. SCHMALL, New York City.

The proposer evidently meant that the three lines are parallel.

(i) Let x , y , and z be the three parallel lines in order. At any point, M , on the line y , construct the angles AMC and BMC each equal to 60° (Fig. 1). Draw the line AB , and, about the triangle AMB , circumscribe the circle ABC .

Draw the lines AC and BC . Then, the triangle ABC is equilateral as is easily seen and, therefore, satisfies the condition of the problem.

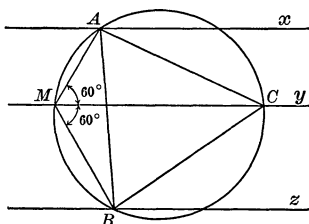


Fig. 1.

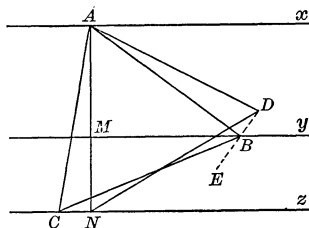


Fig. 2.

(ii) We may also construct the triangle as follows: Through the point, M (Fig. 2), draw AN perpendicular to the line y , and on AN as a side construct the equilateral triangle AND . Through D , draw ED perpendicular to AD , meeting the line y in B . Draw the line AB . Then draw AC , making the angle NAC equal to the angle BAD . Draw BC . Then the triangle ABC is equilateral, and has its vertices on the three given lines as is required. For, the two right triangles BAD and CAN are congruent, having, by construction, a side and an angle of one made equal to a side and an angle of the other. Hence, side AC equals side AB . Angle CAB equals angle $NAD = 60^\circ$. Hence, angles ACB and ABC are equal and the triangle is equilateral.

II. SOLUTION BY NATHAN ALTSHILLER, University of Washington.

Let us generalize the problem in the following way:

Construct a triangle ABC so that its vertices A, B, C shall lie respectively on three given lines p, q, r , while its angles α, β, γ shall have each a given value.

The condition that the triangle shall have three given angles is equivalent to the condition that one of its angles, say α , shall have a given value, and that the ratio AB/AC , of the sides AB and AC including this angle, shall be equal to a given quantity k .

Let D be the foot of the perpendicular AD dropped from A upon the line r . Rotate the triangle ACD bodily about the point A into the position $AC'D'$ such that the line AC shall take the direction of the line AB . The angle DAD' is equal to the angle $CAB = \alpha$.

The perpendicular BE dropped from B upon AD' , determines on AD' a point E such that

$$\frac{AE}{AD'} = \frac{AB}{AC'} = \frac{AB}{AC} = k.$$

This leads to the following method of solving the problem:

From any point A on p drop the perpendicular AD upon r and construct the line AD' making with AD an angle equal to the given angle α . On AD' take a

segment $AD' = AD$ and determine on it the point E so that $\frac{AE}{AD'} = k$. The perpendicular EB to AD' at the point E will meet q in a point B which is the second vertex, besides A , of the required triangle. The third vertex C is easily found.

This construction is applicable whether the given lines are concurrent, parallel, or form a triangle.

To satisfy the conditions of the generalized problem, it is necessary to exercise care in effecting the rotation in the proper sense. If the problem should read that ABC is to be similar to a given triangle without specifying which angle shall have its vertex on which of the given lines, the point A would be the origin of three triangles each solving the problem, but differently situated. The three triangles coincide, if the angles of ABC are to be equal to each other, as in the case of the proposed problem.

The particular case was also solved by DANIEL KRETH, ELIZABETH B. DAVIS, CLIFFORD N. MILLS, S. A. JOFFE, ELMER SCHUYLER, J. W. CLAWSON, and the PROPOSER.

455. Proposed by R. P. BAKER, University of Iowa.

Find the minimum triangle of assigned angles inscribed in a given triangle.

SOLUTION BY ROGER A. JOHNSON, Western Reserve University.

By the well-known theorem of MIQUEL (McClelland, *Geometry of the Circle*, pp. 40–42), if a triangle PQR be inscribed in a given triangle ABC so as to be directly similar, i. e., without being turned over, to a given triangle; then the circles AQR , BRP , CPQ pass through a point O , which, moreover, is fixed for all positions of the inscribed triangle; further, if P , Q , R lie on the sides of the triangle, and not on their extensions, $\angle OPB = \angle OQC = \angle ORA$. If from the point O , lines OP_1 , OQ_1 , OR_1 be drawn, making equal angles with the sides, their extremities P , Q , R , will form a triangle similar to PQR .

It follows that if the point O be known, we shall obtain the minimum triangle by taking the pedal triangle; that is to say, the triangle whose vertices are the feet of the perpendiculars from O to the sides of ABC .

First inscribe in ABC any triangle of the given species. The following, due to Petersen, is, perhaps, the simplest. Take Y on AC and Z on AB , at random. On YZ construct a triangle XYZ of the given type and in the desired position. Let AX cut BC at P . Through P draw PQ and PR parallel to XY and XZ , respectively. Then QR will be parallel to YZ and PQR will be an inscribed triangle of the desired type.

Now draw the circles AQR , BRP , CPQ , intersecting again at O . From O , drop perpendiculars OL , OM , ON , to the sides. Then LMN is the required triangle.

Also solved by C. N. SCHMALL, FRANK IRWIN, and J. W. CLAWSON.

CALCULUS.

368. Proposed by PAUL CAPRON, Annapolis, Maryland.

Develop $\log_{10} x/\sin x$ and $\log_{10} \tan x/x$ to three terms as functions of $\log_{10} \sec x$, showing that